Upper bound on Generalization Error

**Theorem (T. Zhang [3]):** If $O(n) > 1$, OMP output after $s = O(s_r) \log(n)$ iterations, then $n \leq k$.

We adopt the ad-hoc to improve the upper bound.

### Generalization error upper bound

**Theorem:**

After $s = O(s_r) \log(n)$ iterations, with high probability, for any $x \in S$

\[
\|\bar{x}_n - x\|_2 \leq \left(\frac{M}{s_r} + \frac{1}{s}\right) \sqrt{\frac{s_r \log(n)}{n}}
\]

**Remarks:**

- The assumption $n \geq k$ is too strong in most cases.
- The parameter $s_r$ is order of the design matrix and the sub-
- The only limits for recovery results is for LASSO under BCS, that probable similar result [3]

**Results for Gaussian ensemble**

- When norm of $A$ is bounded, $s = O(k)$ holds.

- Generalization error $\|\bar{x}_n - x\|_2 \leq \sqrt{\frac{M}{s_r} \log(n)}$

- Support recovery: $\|\bar{x}_n - x\|_1 \leq \frac{1}{s} \sqrt{\frac{M}{s_r} \log(n)}$

### Support Recovery upper bound

**Theorem:**

After $s = O(s_r) \log(n)$ iterations, with high probability, for any $x \in S$

\[
\|\bar{x}_n - x\|_1 \leq \frac{1}{s} \sqrt{\frac{M}{s_r} \log(n)}
\]

### Large in objective

**Theorem:**

After $s = O(s_r) \log(n)$ iterations, with high probability, for any $x \in S$

\[
\|\bar{x}_n - x\|_2 \leq \left(\frac{M}{s_r} + \frac{1}{s}\right) \sqrt{\frac{s_r \log(n)}{n}}
\]

**Remarks:**

- The objective function can be further bounded via distortion can be too large.

### Lower bound results

**Theorem:**

- When norm of $A$ is bounded, $s = O(k)$ holds.

- Generalization error $\|\bar{x}_n - x\|_2 \leq \sqrt{\frac{M}{s_r} \log(n)}$

- Support recovery: $\|\bar{x}_n - x\|_1 \leq \frac{1}{s} \sqrt{\frac{M}{s_r} \log(n)}$

### References


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